A New Application of Sawi Transform on Some Partial Differential Equations

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Abstract:
In this paper a new integral transform namely " Sawi transform " was applied to solve some partial differential equations.

Key words: Sawi transform , partialdifferential equations.

1. **Introduction**:Integral transform methods (Laplace transform, Fourier transform ,Mahgoub transform, Kamal transform ,Aboodh transform,Mohand transform, Elzaki transform, Shehu transform, and Sumudu transorm) are suitable mathematical tools for solving advanced problems of sciences and engineering which are expressible in terms of differential equations, delay differential equations, system of differential equations, partial differential equations, integral equations, system of integral equations, partial integro-differential equations and integro- differential equations.Sawi transform was introduced by Mohand Mahgoub to facilitate the process of solving ordinary and partial differential equations in the time domain. The aim of this study is to show the applicability of this interesting new transform "Sawi transform " and its ability to solve some partial differential equations.

2. Definitions and Basic Results.

2.1. Definition of Sawi transform [1,2,3,4]

A new transform called the Sawi transform defined for function of exponential order we consider functions in the set A defined by:

$$A = \left\{ f(t): \exists M, K_1, K_2 > 0, |f(t)| < M e^{\frac{|t|}{k_j}}, if\ t \in (-1)^j x\ [0, \infty) \right\} \ (1)$$

For a given function in the set A, the constant M must be finite number, K_1, K_2 may be finite or infinite.

Sawi transform denoted by the operator S (.) defined by the integral equations

$$S\,[f(t)] = R(v) = \frac{1}{v^2}\int_0^\infty f(t) e^{\frac{-t}{v}}\, dt,\ t \geq 0\ , K_1 \leq v \leq K_2 \ (2)$$

The variable v in this transform used to factor the variable t in the argument of the function f.

2.2 Some properties of Sawi transform.

2.2.1 Linearity property [1]

If $S\,[f(t)] = H(v)$ and $S\,[g(t)] = I(v)$ then

$$S\,[af\,(t) + bg(t)] = aH(v) + bI(v) \ (3)$$

Where a, b are arbitrary constants.

2.2.2 Change of scale property [1]

If $S\,[f(t)] = R(v)$ then

$$S\,[f(at)] = aR(av) (4)$$

2.2.3 Sawi transform of the derivatives[1]

If $S[f(t)] = R(v)$ then

(i) $$S[f'(t)] = \frac{R(v)}{v} - \frac{f(0)}{v^2} \ (5)$$

(ii) $$S[f''(t)] = \frac{R(v)}{v^2} - \frac{f(0)}{v^3} - \frac{f'(0)}{v^2} (6)$$

(iii) $$S\left[f^{(n)}(t)\right] = \frac{R(v)}{v^n} - \frac{f(0)}{v^{n+1}} - \frac{f'(0)}{v^n} - \cdots - \frac{f^{(n-1)}(0)}{v^2} (7)$$

2.2.4 Convolution theorem for Sawi transforms[1]

If $S\,[f(t)] = H(v)$ and $S\,[g(t)] = I(v)$ then

$$S\,[f(t) * g(t)] = v^2 S\,[f(t)] S[g(t)] = v^2 H(v) I(v) (8)$$

Where $f(t) * g(t) = \int_0^t f(x)g(t-x)dx$

2.2.5 Translation property [2]

If $S\,[f(t)] = R(v)$, then

$$S\,[e^{kt}\,f(t)] = (1/(1-kv)^2)R(v/(1-kv\,))(9)$$

Where k is an arbitrary constant

2.2.6 Sawi transform of the partial derivatives

If $S\,[u(x,t)] = \Phi(x,v)$ then

(i) $S\left[\frac{\partial u(x,t)}{\partial t}\right] = v^{-1}\Phi(x,v) - v^{-2}u(x,0)$

(10)

(ii) $S\left[\frac{\partial^2 u(x,t)}{\partial t^2}\right] = v^{-2}\Phi(x,v) - v^{-3}u(x,0) - v^{-2}\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}$ (11)

(iii) $S\left[\frac{\partial u(x,t)}{\partial x}\right] = \frac{d}{dx}\Phi(x,v)$ (12)

(iv) $S\left[\frac{\partial^2 u(x,t)}{\partial x^2}\right] = \frac{d^2}{dx^2}\Phi(x,v)$ (13)

(v) $S\left[\frac{\partial^n u(x,t)}{\partial x^n}\right] = \frac{d^n}{dx^n}\Phi(x,v)$ (14)

Proof:

(i) $S\left[\frac{\partial u(x,t)}{\partial t}\right]$

$$= v^{-2}\int_0^\infty \frac{\partial u(x,t)}{\partial t} e^{\frac{-t}{v}} dt = v^{-2}\lim_{A\to\infty} \int_0^A e^{\frac{-t}{v}} \frac{\partial u(x,t)}{\partial t}\, dt$$

$$= v^{-2}\lim_{A\to\infty}[e^{\frac{-t}{v}}\, u(x,t)]_0^A + v^{-3}\int_0^A e^{\frac{-t}{v}}\, u(x,t)dt$$

$$= -v^{-2}u(x,0) + v^{-1}\,\Phi(x,v)$$

(ii) Let $\frac{\partial u(x,t)}{\partial t} = w(x,t) \Rightarrow$

$$S\left[\frac{\partial^2 u(x,t)}{\partial t^2}\right] = S\left[\frac{\partial w(x,t)}{\partial t}\right] = v^{-1}\,S\,[w(x,t)] - v^{-2}\, w(x,0)$$

$$= v^{-2}\,\Phi(x,v) - v^{-3}u(x,0) - v^{-2}\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0}$$

(iii, ivand v) Using the Leibniz rule, easily we get

$$S\left[\frac{\partial u(x,t)}{\partial x}\right] = \frac{d}{dx}\Phi(x,v)$$

$$S\left[\frac{\partial^2 u(x,t)}{\partial x^2}\right] = \frac{d^2}{dx^2}\Phi(x,v)$$

$S\left[\frac{\partial^n u(x,t)}{\partial x^n}\right] = \frac{d^n}{dx^n}\Phi(x,v)$

3. Sawi transform of some elementary functions [1]

S.N	$f(t)$	$S\,[f(t)] = R(v)$
1.	1	v^{-1}
2.	t	1
3.	t^2	$2!\,v$
4.	$t^n, n \in N$	$n!\,v^{n-1}$
5.	$t^n, n > -1$	$\Gamma(n+1)v^{n-1}$
6.	e^{at}	$\frac{1}{v(1-av)}$
7.	$\sin at$	$\frac{a}{1+a^2v^2}$
8.	$\cos at$	$\frac{1}{v(1+a^2v^2)}$
9.	$\sinh at$	$\frac{a}{1-a^2v^2}$
10.	$\cosh at$	$\frac{1}{v(1-a^2v^2)}$

4. Applications [5]

Example 4.1

Solve the following initial value problem using the Sawi transform

$u_x - 2u_t = u\,,\ \ x,t > 0$ (15)

$u(x,0) = e^{-3x}$ (16)

Solution: Taking theSawi transform of both sides of the givenpartial differential equation, we have

$\frac{d\alpha}{dx} + (-1 - 2v^{-1})\alpha = -2v^{-2}\,e^{-3x}$ (17)

Where$\alpha = S\,[u]$, thus the solution of the first orderpartial differential equation reduces to the solution of firstorder ordinary differential equation given by

$\frac{d\,S[u]}{dx} + (-1 - 2v^{-1})S\,[u] = -2v^{-2}\,e^{-3x}$ (18)

The general solution of equation (18) is found to be

$S\,[u] = \frac{1}{v(2v+1)}e^{-3x} + c\;\;e^{\left(1+\frac{2}{v}\right)x}$ (19)

Since $S\,[u]$is bounded, c should be zero, and if we take the inverse Sawi transform of both sides of equation (19), then the solution of equation (15) with initialcondition(16) is:

$u\;\;(x,t) = e^{-3x-2t}$ (20)

Example 4.2

Using the Sawi transform, solve the following initial boundaryvalue problem:

$u_{tt} - u_{xx} = 0,\;\; 0 \le x \le \pi,\;\; t \ge 0$ (21)

$u(x,0) = \sin x,\; u(0,t) = 0,\; u_t\,(x,0) = 0, u(\pi,t) = 0$(22)

Solution: Taking theSawi transform of both sides of the given partial differential equation, we get

$\frac{d^2S[u]}{dx^2} - \frac{1}{v^2}S[u] = \frac{-\sin x}{v^3}$ (23)

This is the second-order ordinary differential equation has the solution in the form:

$S[u] = \frac{\sin x}{v(v^2+1)}$ (24)

Now, if we take the inverse Sawi transform of equation (24), then the particular solution of equation (21) is:

$u(x,t) = \sin x\;\cos t$ (25)

Example 4.3

Solve following partial differential equation using the Sawi transform

〖$u\_(t\,) = u\_xx$, $0 < x < 2,\;\; t > 0$〗_ (26)

$u(0,t) = 0,\; u(2,t) = 0$

$u(x,0) = 3\sin(2\pi x)$

Solution: Taking theSawi transform and applying the initial condition

$$\frac{d^2}{dx^2}S[u] - \frac{1}{v}S[u] = \frac{-1}{v^2}3\sin(2\pi x) \qquad (27)$$

After solving this ordinary differential equation and using the boundary conditions, we get

$$S[u] = \frac{3}{v(1+4\pi^2 v)}\sin(2\pi x) \qquad (28)$$

To find our solution we apply the inverse Sawi transform

$$u(x,t) = S^{-1}\left[\frac{3}{v(1+4\pi^2 v)}\right]\ \sin(2\pi x)$$

$$= 3e^{-4\pi^2 t}\sin(2\pi x)$$

Example 4.4

Solve the following boundary value problem using the Sawi transform

$$u_{tt} = u_{xx} + \sin(\pi x),\ \ 0 < x < 1,\ \ t > 0$$

$$u(x,0) = 0, \qquad u_t\,(x\,,0) = 0$$

$$u(0,t) = 0,\ u(1\,,t) = 0 \qquad (29)$$

Solution: Taking theSawi transform and applying the initial conditions we obtain

$$\frac{d^2}{dx^2}S[u] - \frac{1}{v}S[u] = \frac{-\sin(\pi x)}{v} \qquad (30)$$

The solution of equation (30) is in the following form

$$S[u] = \frac{v^2}{v(1+\pi^2 v^2)}\sin(\pi x) \qquad (31)$$

Taking the inverse Sawi transform we have

$$u(x,t) = \frac{1}{\pi^2}[1 - \cos(\pi t)]\ \sin(\pi x) \qquad (32)$$

CONCLUSION

In this paper, we have successfully discussed the Sawi transform for some partial differential equations. The given numerical applications in application section shows the importance of Sawi transform for partial differential equations.

Results show that theSawi transform is very useful integral transform for handling partial differential equations. The scheme defined in this paper can be applied for finding the solution of boundary value problems ofheat conduction problems, vibrating beam problems, and other mathematical physics problems, all the solutions in this paper are satisfied by putting them back in the original equations.

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